

MR1241876 (94h:14044) 14J70 (14N10)

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A quintic hypersurface in $\mathbf{P}^4$ with 130 nodes.

Topology **32** (1993), *no. 4*, 857–864.

An old problem in algebraic geometry is to determine the maximum number $N_n(d)$ of ordinary double points (nodes) a hypersurface of degree d in $\mathbf{P}^n$ can have, for $n \geq 3$. For $n = 3$, one has $N_3(3) = 4$ (the Cayley cubic has four nodes) and $N_3(4) = 16$ (the Kummer surface has 16 nodes). Beauville showed that $N_3(5) = 31$ [A. Beauville, in *Journées de Géométrie Algébrique d'Angers, Juillet 1979/Algebraic Geometry, Angers, 1979*, 207–215, Sijthoff & Noordhoff, Alphen aan den Rijn, 1980; [MR0605342 \(82k:14037\)](#)]. The numbers $N_3(d)$, for $d \geq 6$, are not known.

For $n = 4$, it is known that $N_4(3) = 10$ (realized by the Segre cubic) and that $N_4(4) = 45$ (realized by the Burkhardt quartic). The best known upper bounds for $N_n(d)$ ($n \geq 4$) are the so-called spectral bounds obtained by Varchenko—in particular, he obtained $N_4(5) \leq 135$.

In the present paper, the author gives a very nice construction of a quintic threefold with 130 nodes—thus increasing the lower bound for $N_4(5)$ from 126 (obtained by Hirzebruch) to 130.

Reviewed by [Ragni Piene](#)

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